

Excited State Spectroscopy from Lattice QCD

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Spectroscopy

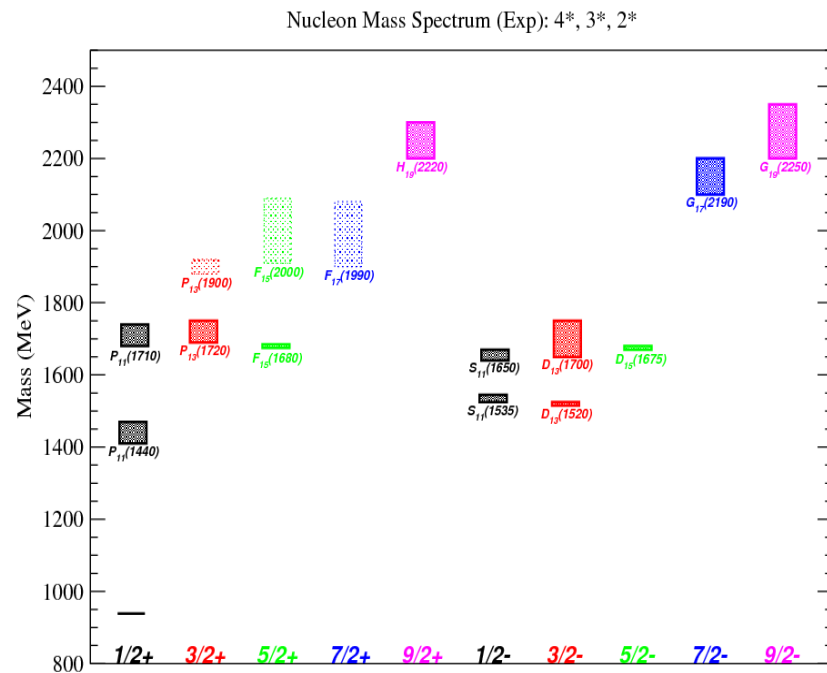
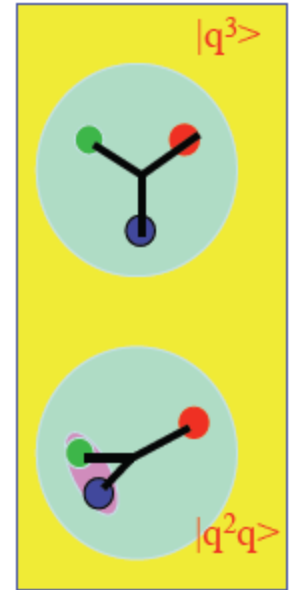
Spectroscopy reveals fundamental aspects of hadronic physics

- Essential degrees of freedom?
- Gluonic excitations in mesons - exotic states of matter?
- Status
 - Can extract excited hadron energies & identify spins,
 - Pursuing full QCD calculations with realistic quark masses.
- New spectroscopy programs world-wide
 - E.g., BES III, GSI/Panda
 - Crucial complement to 12 GeV program at JLab.
 - Excited nucleon spectroscopy (JLab)
 - JLab GlueX: search for gluonic excitations.

Baryon Spectrum

"Missing resonance problem"

- What are collective modes?
- What is the structure of the states?
 - Major focus of (and motivation for) JLab Hall B
 - Not resolved experimentally @ 6GeV



Nucleon spectrum

PDG uncertainty on B-W mass

Lattice QCD

Lattice QCD on anisotropic lattices

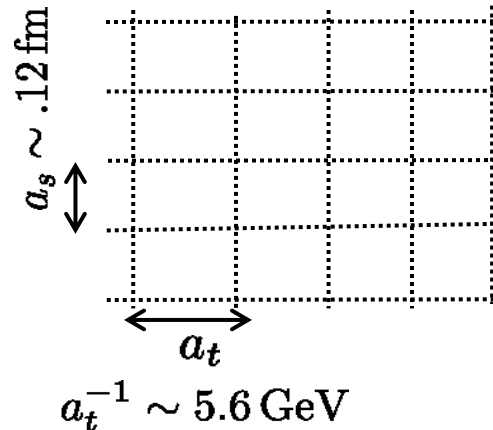
0810.3588

$$N_f = 2 + 1 \text{ (u,d + s)}$$

$$V_s \sim (2.0)^3 \text{ fm}^3, (2.4)^3 \text{ fm}^3, (2.9)^3 \text{ fm}^3, (3.8)^3 \text{ fm}^3$$

$$m_\pi \sim 700, 720, 450, 400, 230 \text{ MeV}$$

$$a_s \sim 0.12 \text{ fm}, (a_t)^{-1} \sim 5.6 \text{ GeV}$$



$$\psi(x) \rightarrow \psi_x$$

$$A_\mu(x) \rightarrow U_{x,\mu} = e^{-aA_{x\mu}}$$

Improved (distilled) operator technology with many operators

0909.0200, 1004.4930

Spectrum from variational method

Two-point correlator

$$C(t) = \langle 0 | \Phi'(t) \Phi(0) | 0 \rangle$$

$$C(t) = \sum_{\mathbf{n}} e^{-E_{\mathbf{n}} t} \langle 0 | \Phi'(0) | \mathbf{n} \rangle \langle \mathbf{n} | \Phi(0) | 0 \rangle$$

Matrix of correlators

$$C(t) = \begin{bmatrix} \langle 0 | \Phi_1(t) \Phi_1(0) | 0 \rangle & \langle 0 | \Phi_1(t) \Phi_2(0) | 0 \rangle & \dots \\ \langle 0 | \Phi_2(t) \Phi_1(0) | 0 \rangle & \langle 0 | \Phi_2(t) \Phi_2(0) | 0 \rangle & \dots \\ \vdots & & \ddots \end{bmatrix}$$

Diagonalize:

eigenvalues \rightarrow spectrum

eigenvectors \rightarrow wave function overlaps

Benefit: orthogonality for near degenerate states

Light quark baryons in $SU(6)$

Conventional non-relativistic construction:

6 quark states in $SU(6)$

$$u_{\uparrow}, u_{\downarrow}, d_{\uparrow}, d_{\downarrow}, s_{\uparrow}, s_{\downarrow}$$

$$SU(6) \subseteq SU(3)_{\text{Flavor}} \otimes SU(2)_{\text{Spin}}$$

Baryons

$$\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6} = \mathbf{56}_S \oplus \mathbf{70}_{MS} \oplus \mathbf{70}_{MA} \oplus \mathbf{20}_A$$

$$\text{Symmetric} : (\mathbf{10}, \mathbf{4}) \quad + (\mathbf{8}, \mathbf{2}) \quad = \mathbf{56}$$

$$\text{Mixed} : (\mathbf{10}, \mathbf{2}) + (\mathbf{8}, \mathbf{4}) + (\mathbf{8}, \mathbf{2}) + (\mathbf{1}, \mathbf{2}) \quad = \mathbf{70}$$

$$\text{Antisymmetric} : \quad (\mathbf{8}, \mathbf{2}) \quad + (\mathbf{1}, \mathbf{4}) = \mathbf{20}$$

Relativistic operator construction: $SU(12)$

Relativistic construction: 3 Flavors with upper/lower components

$$SU(12) \subseteq SU(3)_{\text{Flavor}} \otimes \underbrace{\left[SU(2)_{\text{Upper/lower}} \otimes SU(2)_{\text{Spin}} \right]}_{\text{Dirac}}$$

Times space
(derivatives)

$$\text{Op}_S \leftarrow \text{Derivative}_M \otimes \left[\text{Flavor}_M \otimes \text{Dirac}_M \right]_M$$

Color contraction is Antisymmetric \rightarrow Totally antisymmetric operators

More operators than $SU(6)$: mixes orbital ang. momentum & Dirac spin

Orbital angular momentum via derivatives

Couple derivatives onto single-site spinors:
Enough D's – build any J,M

$$\mathcal{O}^{JM} \leftarrow (CGC's)_{i,j,k} [\vec{D}_M]_i [\vec{D}_M]_j [\Psi_M]_k$$

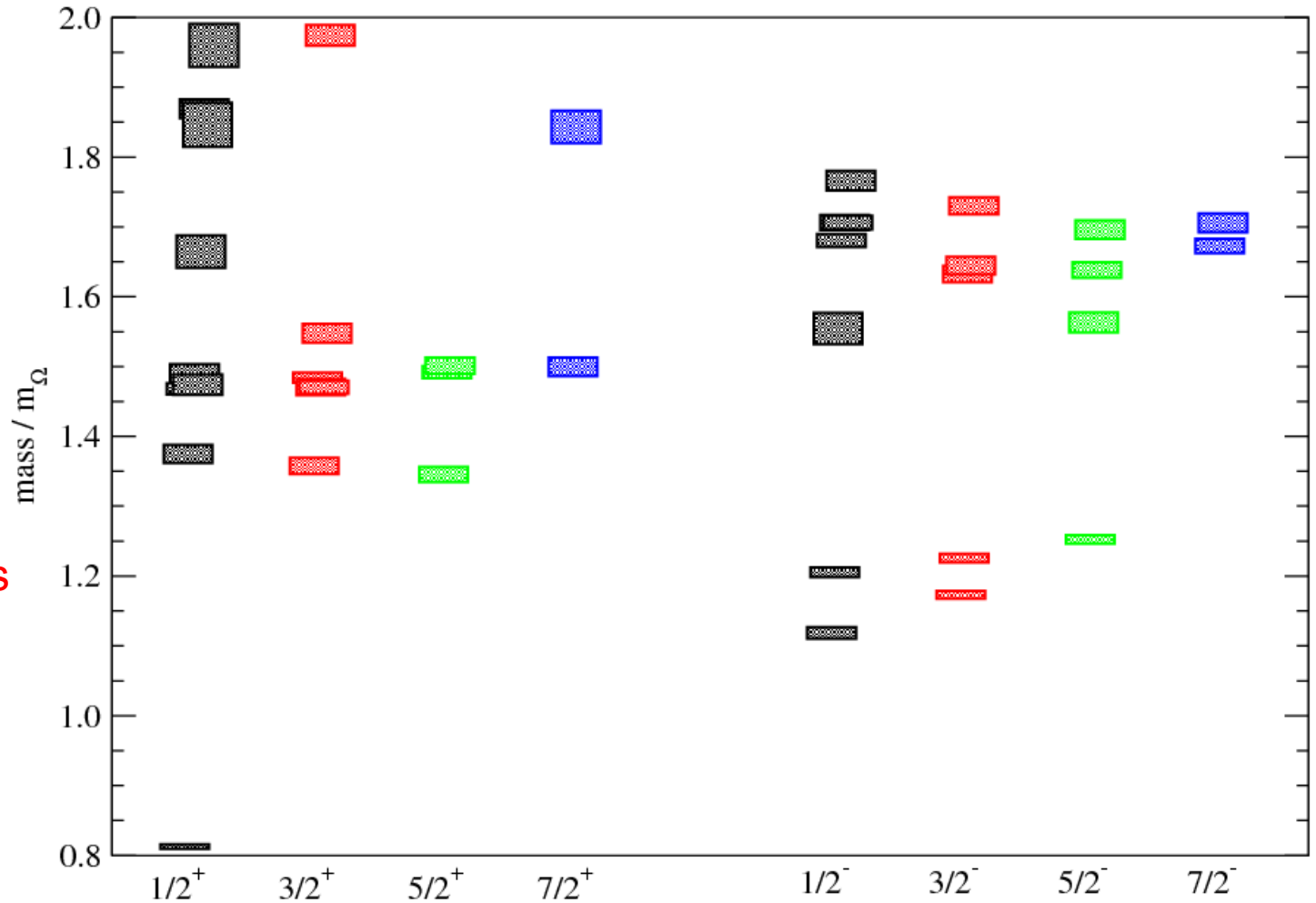
Only using **symmetries** of continuum QCD

Use all possible **operators** up to 2 derivatives (2 units orbital angular momentum)

0905.2160 (PRD), 0909.0200 (PRL), 1004.4930

Spin identified Nucleon spectrum

$m_\pi \sim 520\text{MeV}$

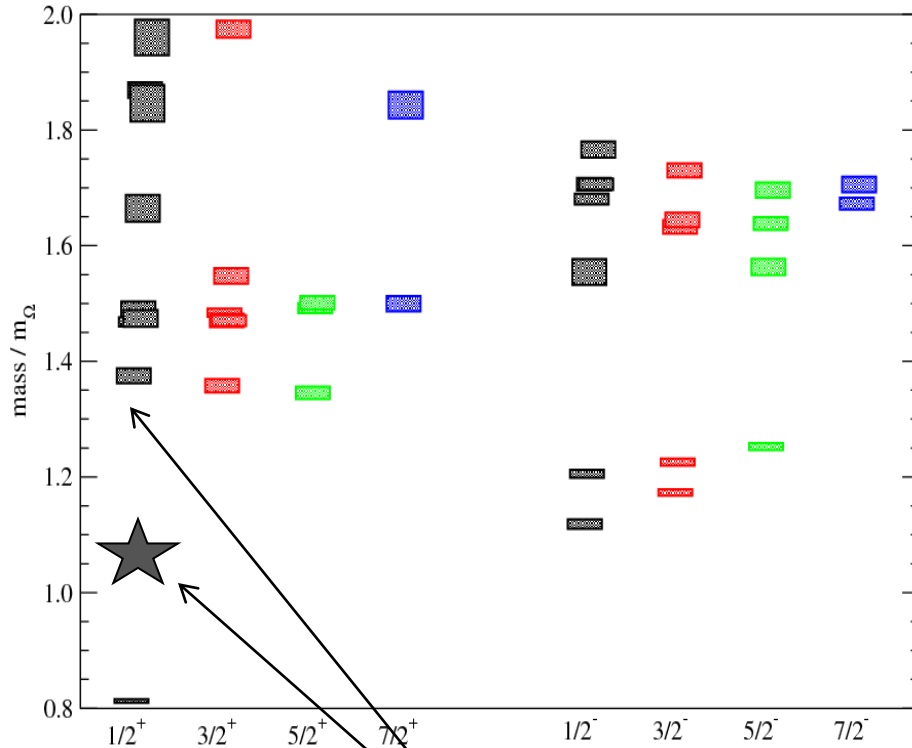


Experimental comparison

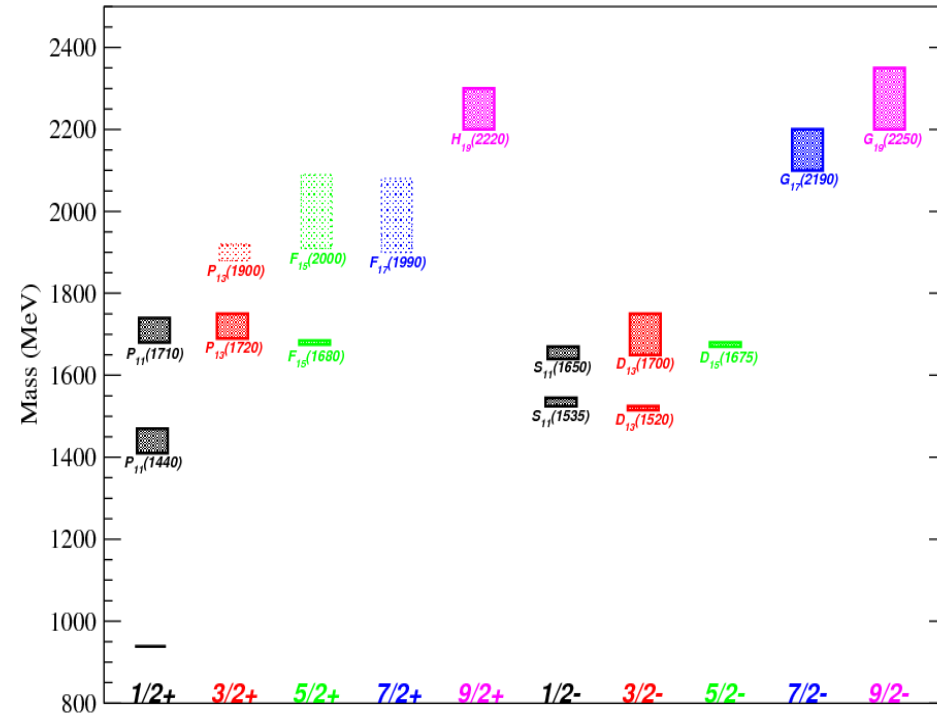
Pattern of states very similar

Nucleons

Nf=2+1, 808, 16^3x128, 7 t0, 250cfs pos parity, 463 neg parity



Nucleon Mass Spectrum (Exp): 4*, 3*, 2*



Where is the "Roper"?

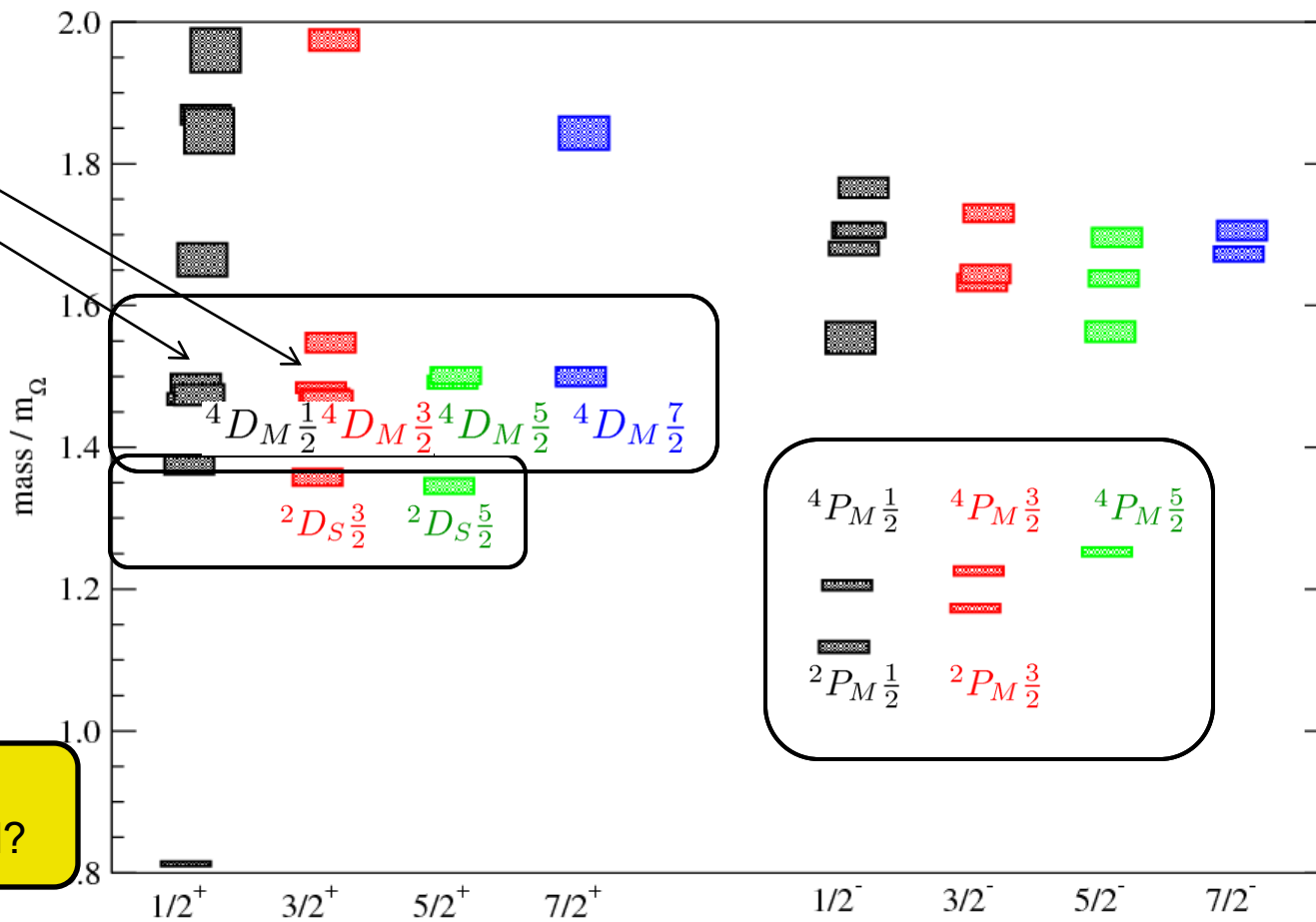
Thresholds & decays: need multi-particle ops

Phenomenology: Nucleon spectrum

Discern structure: wave-function overlaps

$m_\pi \sim 520\text{MeV}$

[20,1+]
P-wave
[70,2+]
D-wave
[56,2+]
D-wave

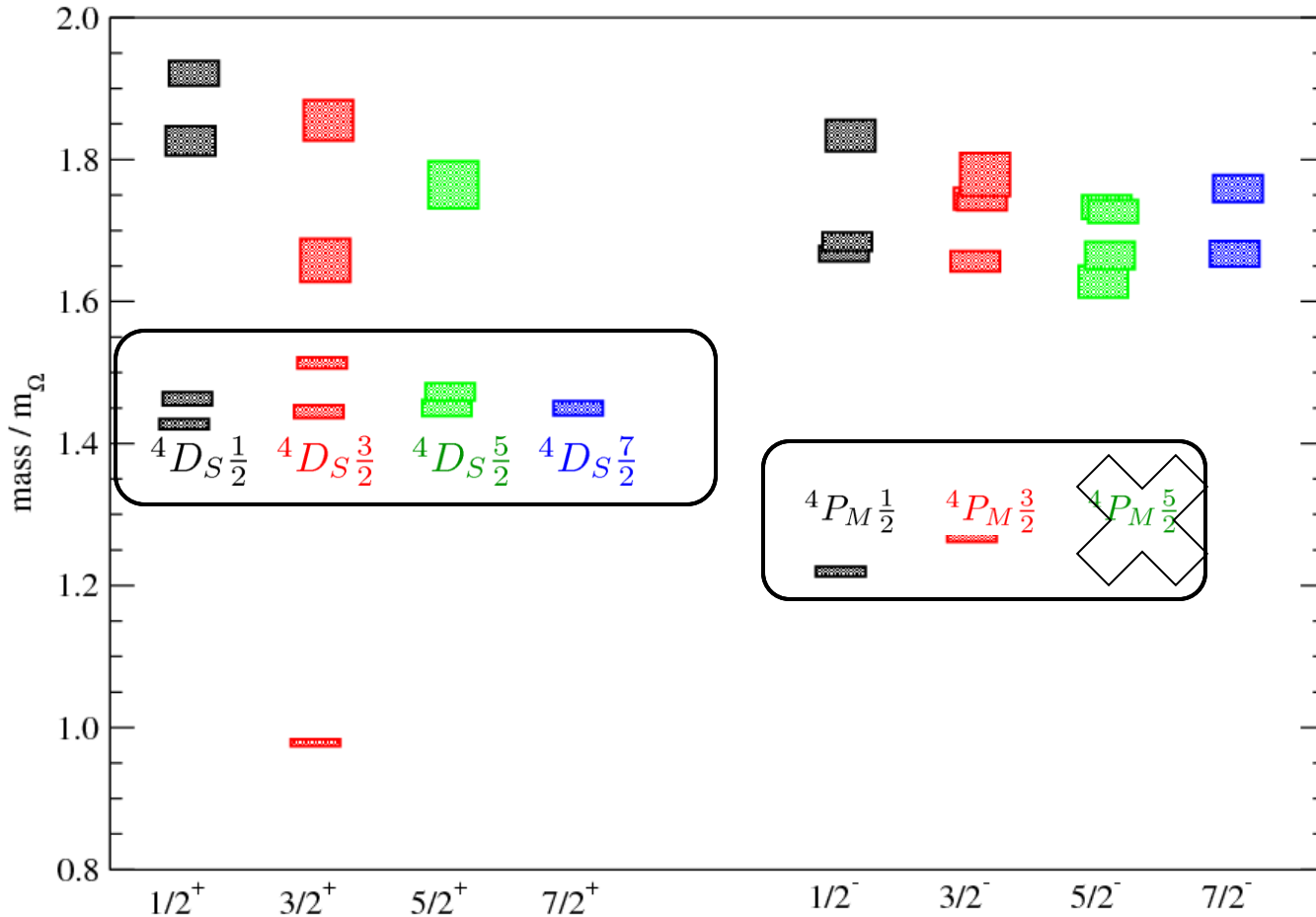


Looks like quark model?

Spin identified Δ spectrum

Spectrum slightly higher than nucleon

[56,2+]
D-wave

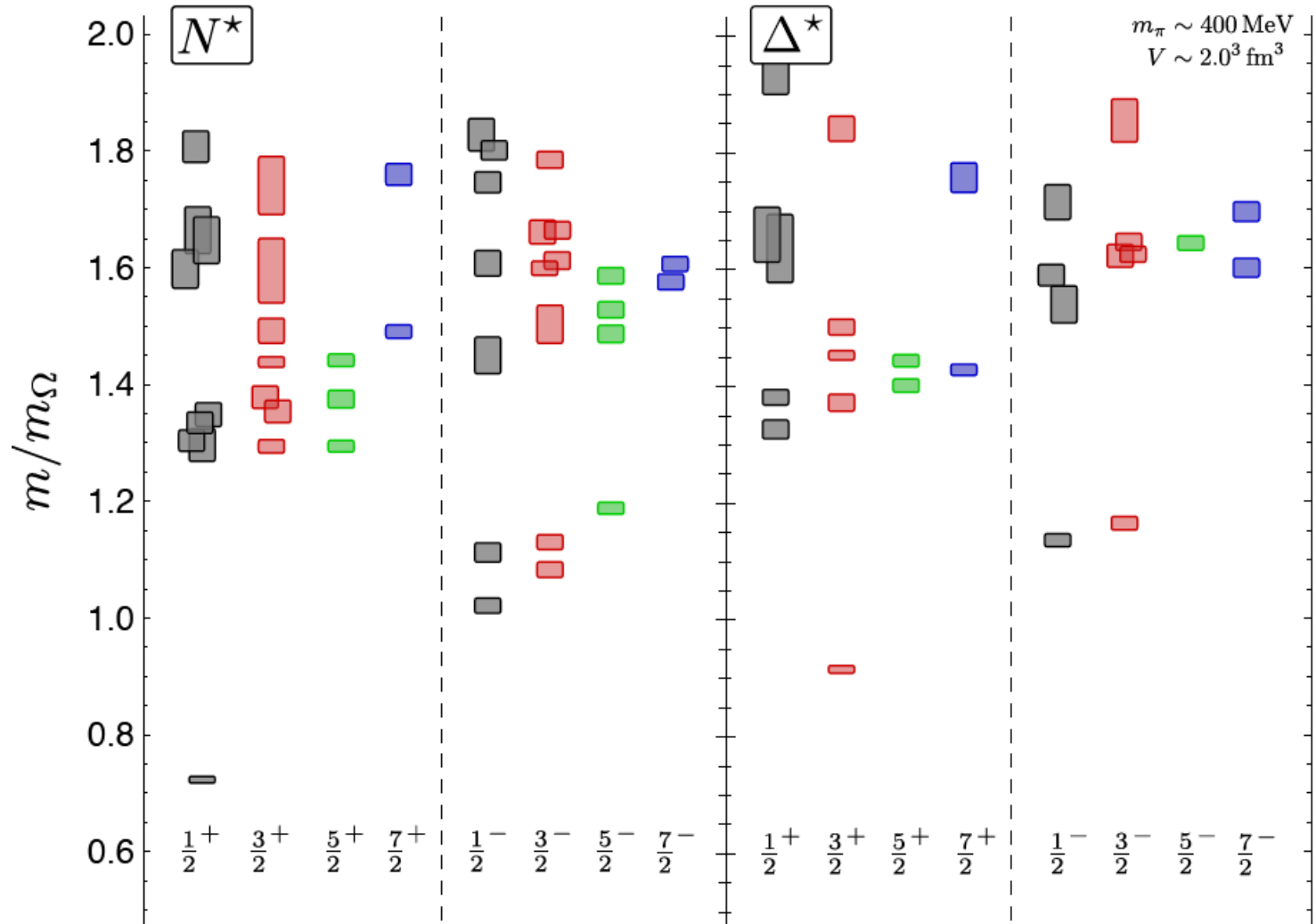


[70,1-]
P-wave

Nucleon & Delta Spectrum

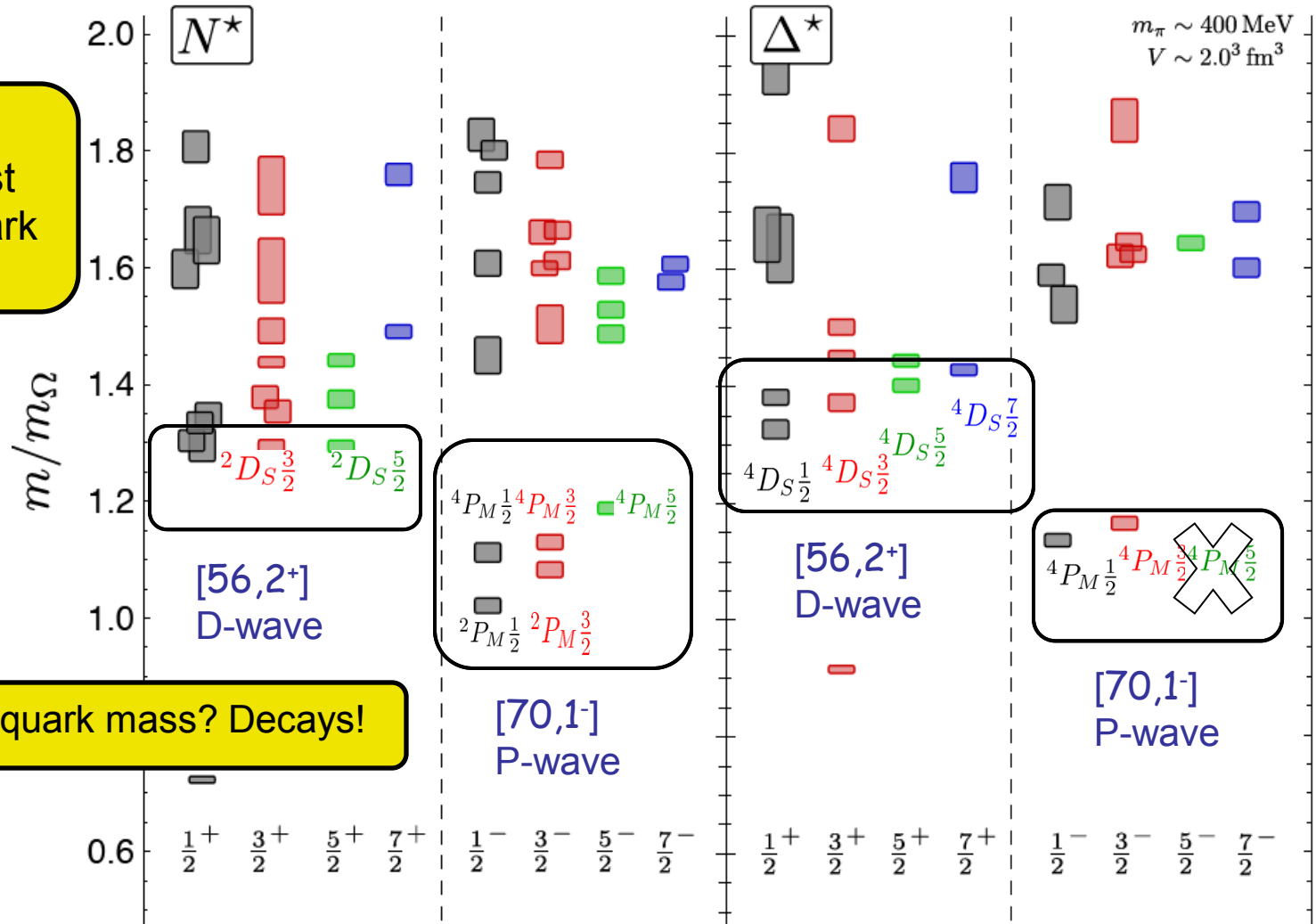
Lighter mass: states spreading/interspersing

$m_\pi \sim 400$ MeV



Nucleon & Delta Spectrum

Suggests spectrum at least as dense as quark model

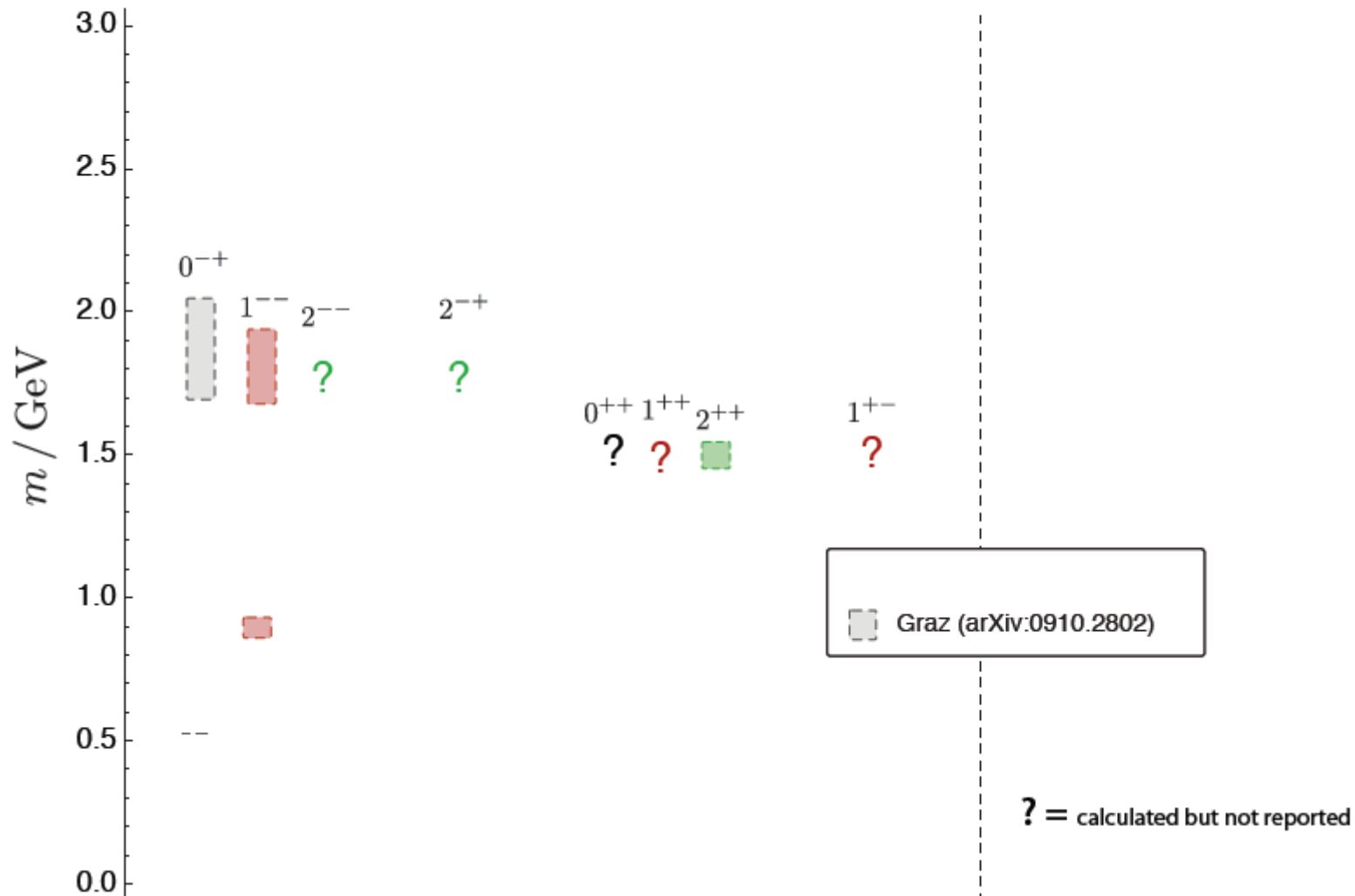


Change at lighter quark mass? Decays!

Isvector Meson Spectrum

the competition

$N_F = 2 (u, d)$ $m_\pi \sim 500 \text{ MeV}$

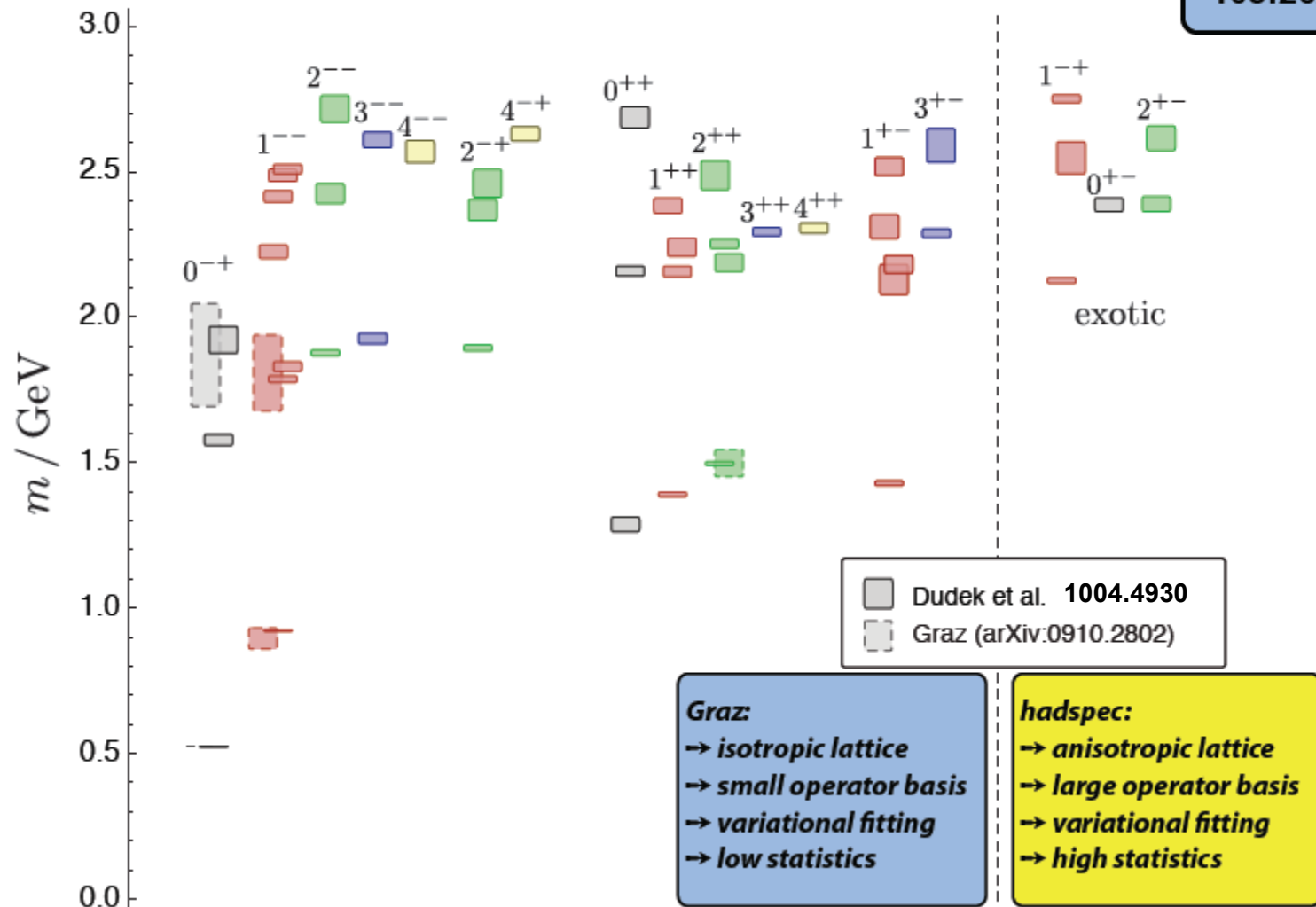


Isvector Meson Spectrum

the competition

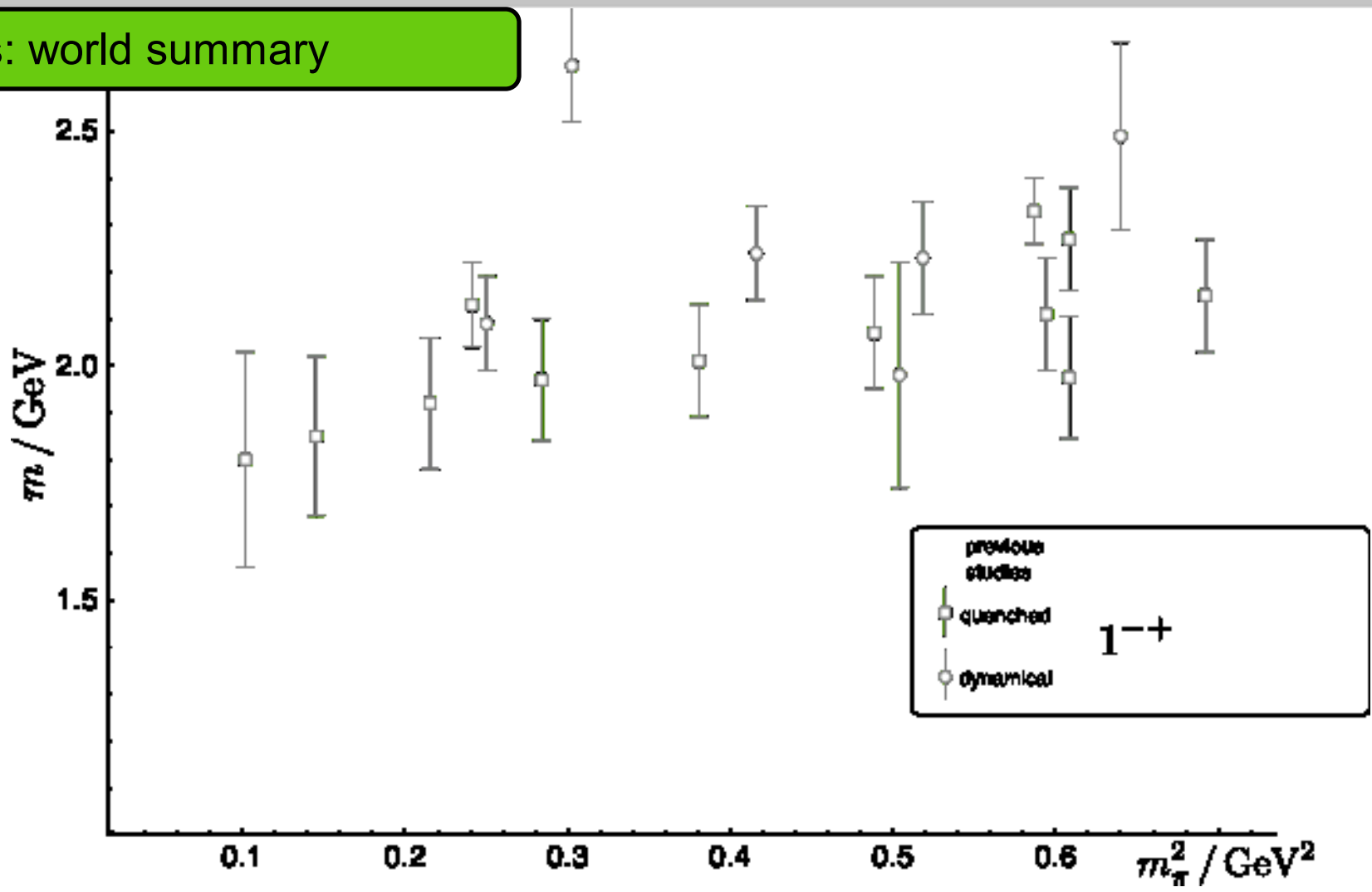
$N_F = 2+1 (u,d,s)$ $m_\pi \sim 520$ MeV $V = 16^3$

PRL
103:262001,2009

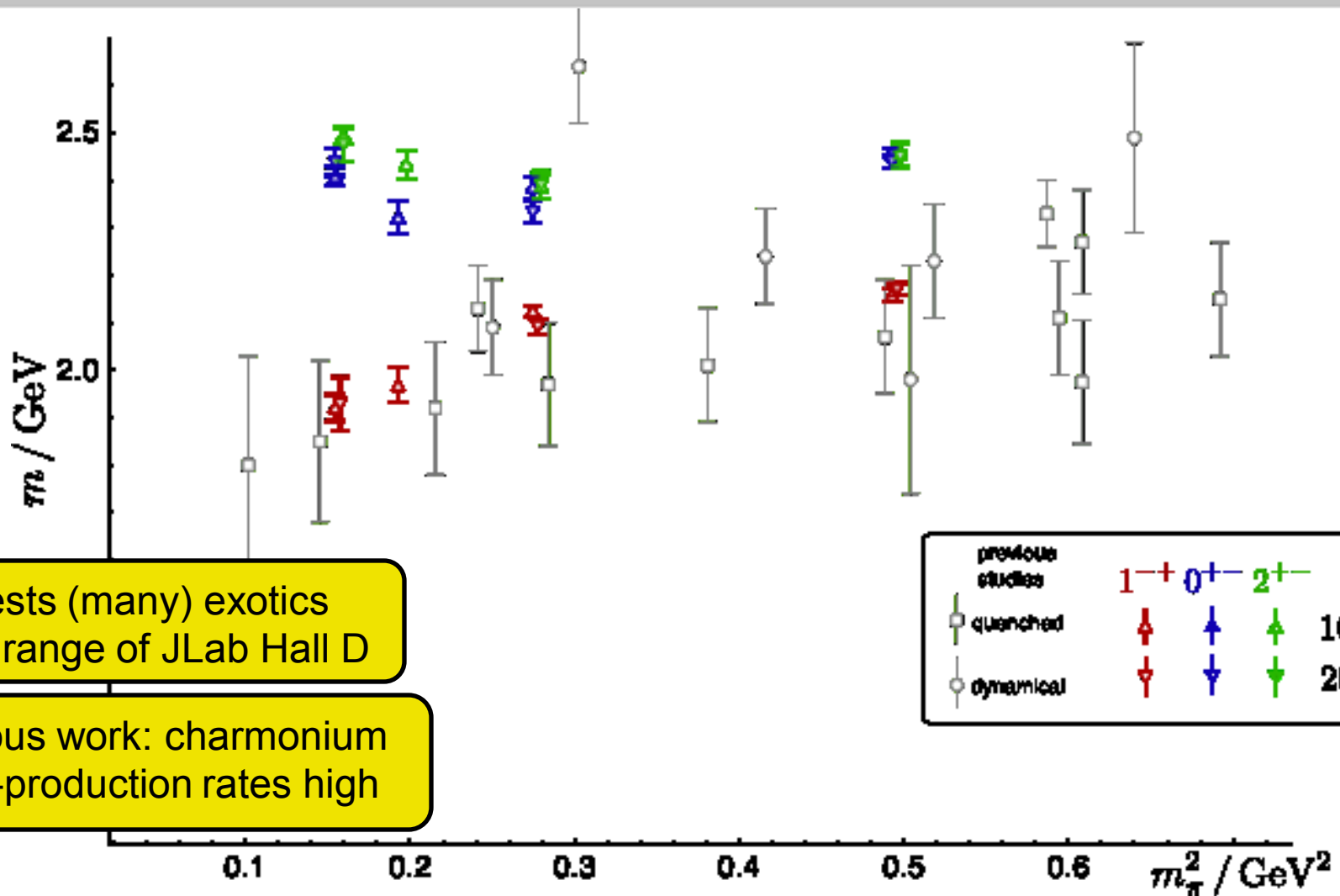


Exotic matter

Exotics: world summary



Exotic matter



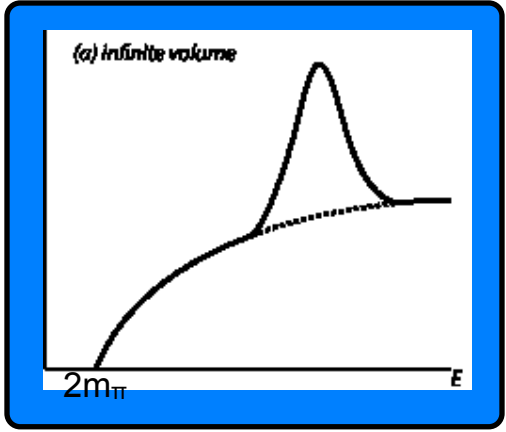
Suggests (many) exotics within range of JLab Hall D

Previous work: charmonium photo-production rates high

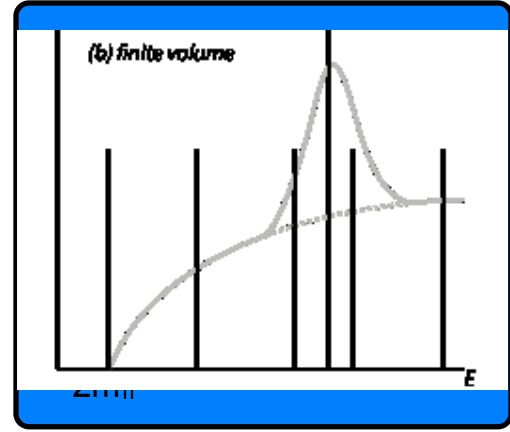
Current work: (strong) decays

Spectrum of finite volume field theory

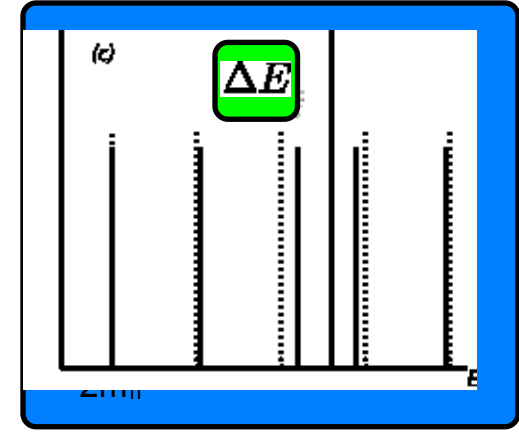
Missing states: “continuum” of multi-particle scattering states



Infinite volume:
continuous spectrum
 $E(p) = 2\sqrt{m_\pi^2 + p^2}$



Finite volume: discrete spectrum



Deviation from (discrete) free energies depends upon interaction - contains information about scattering phase shift

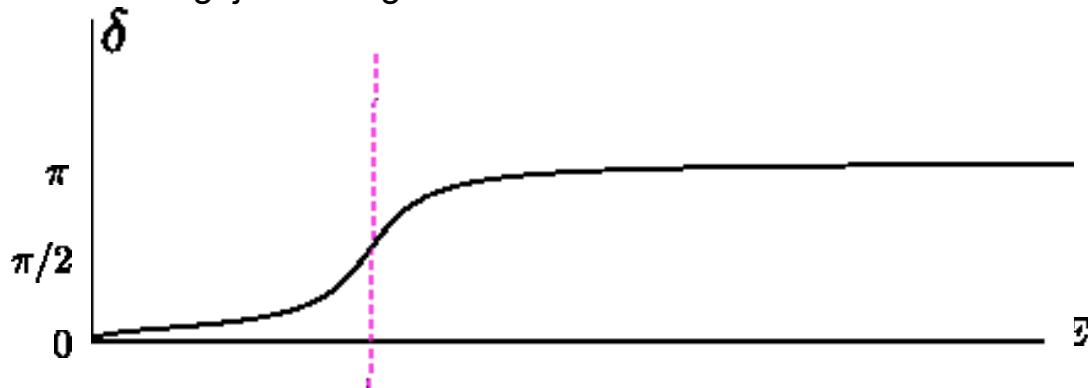
$\Delta E(L) \leftrightarrow \delta(E)$: Lüscher method

Finite volume scattering

Reverse engineer

Use known phase shift - anticipate spectrum

E.g. just a single elastic resonance



e.g.

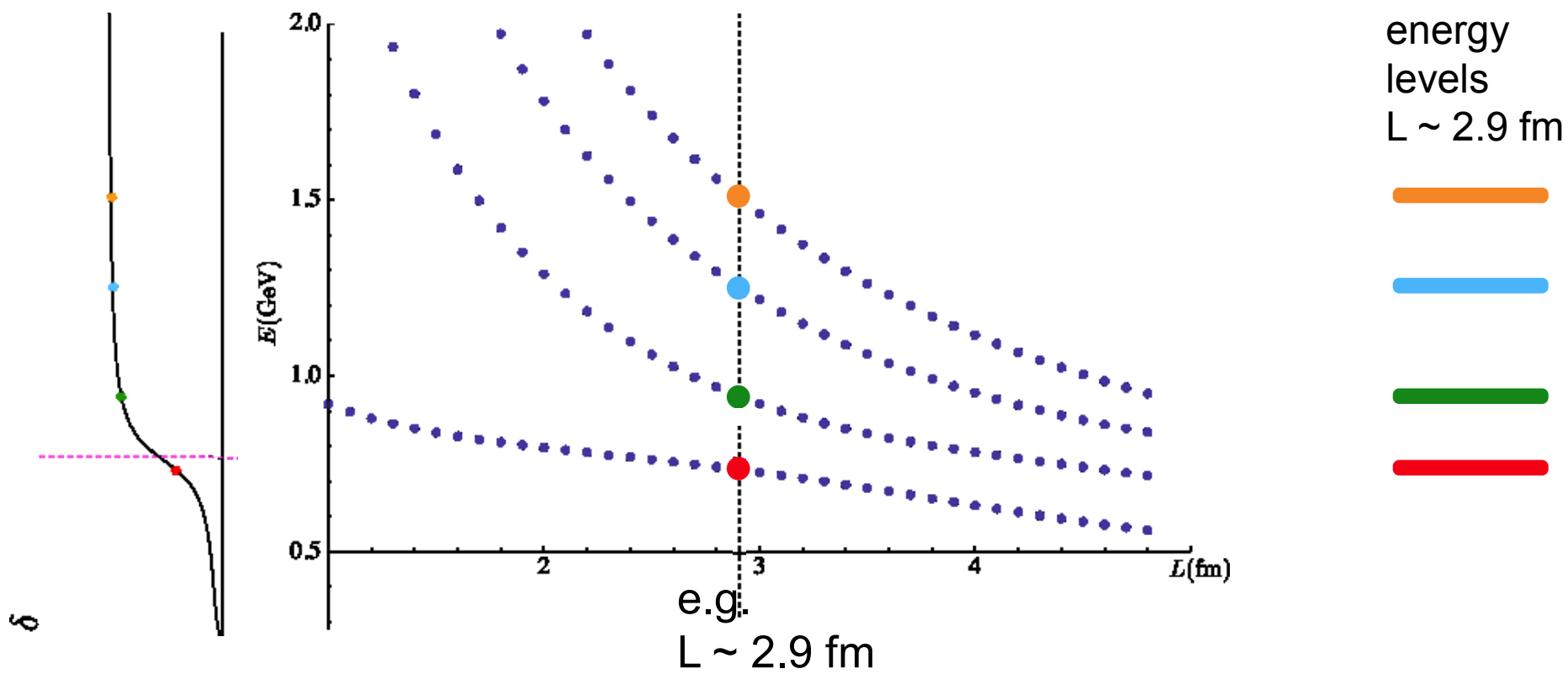
$$\pi\pi \rightarrow \rho \rightarrow \pi\pi$$

$$\pi N \rightarrow \Delta \rightarrow \pi N$$

Lüscher method

- essentially scattering in a periodic cubic box (length L)
- finite volume energy levels $E(\delta, L)$

Finite volume scattering: Lüscher method



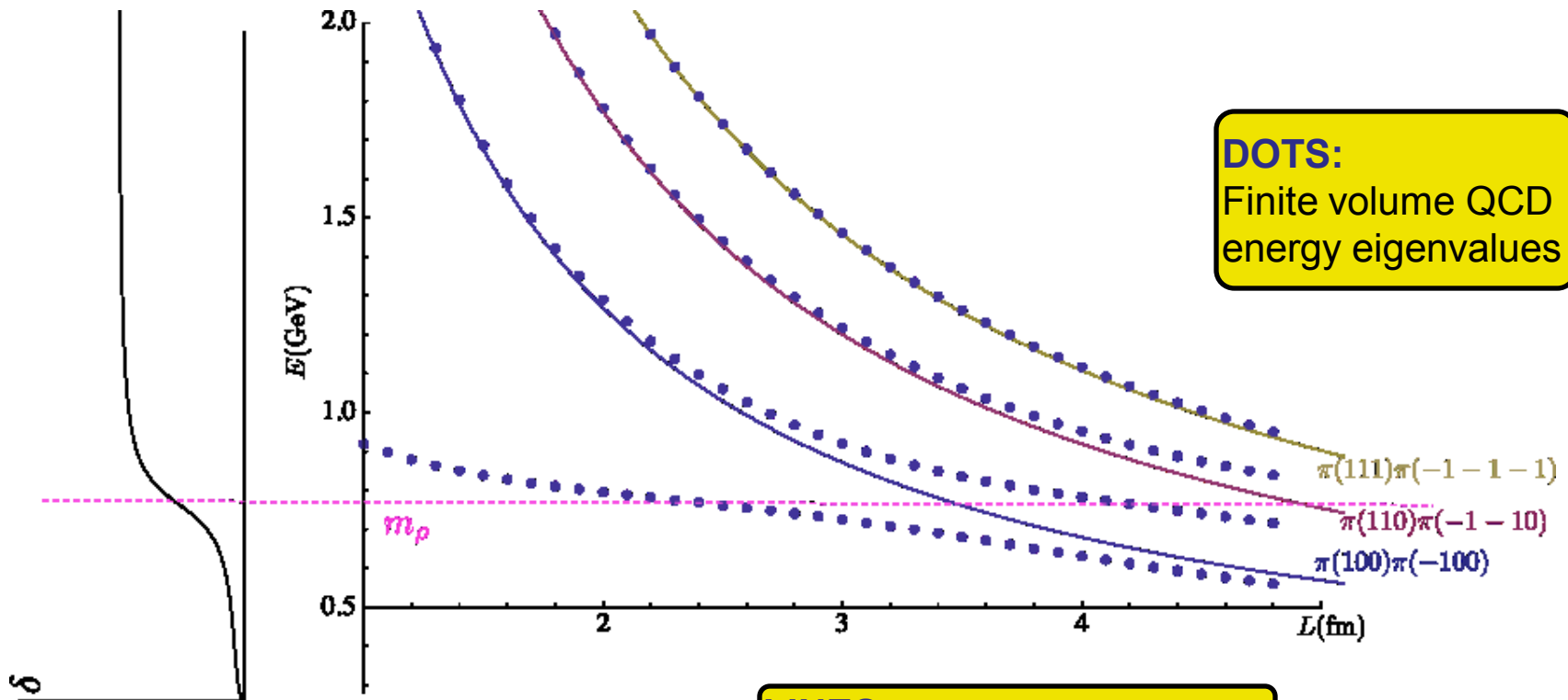
Excited state spectrum at a single volume



Discrete points on the phase shift curve

Do more volumes, get more points

The interpretation



DOTS:
Finite volume QCD energy eigenvalues

“non-interacting basis states”

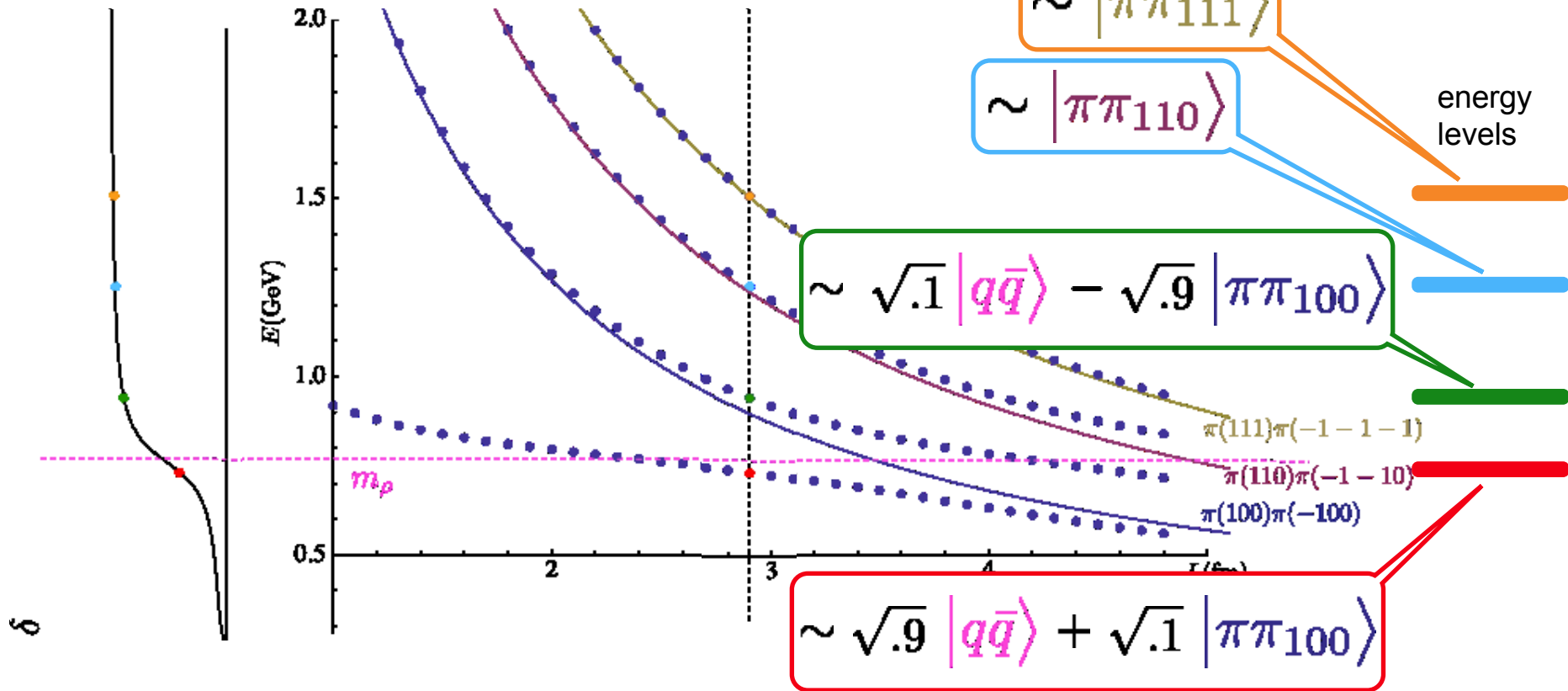
$$|q\bar{q}\rangle \begin{cases} |\pi\pi_{100}\rangle \\ |\pi\pi_{110}\rangle \\ |\pi\pi_{111}\rangle \end{cases}$$

LINES:
Non-interacting two-particle states have known energies

$$E(p) = 2\sqrt{m_\pi^2 + n\left(\frac{2\pi}{L}\right)^2}$$

Level repulsion - just like quantum mechanical pert. theory

The interpretation



Hadronic decays

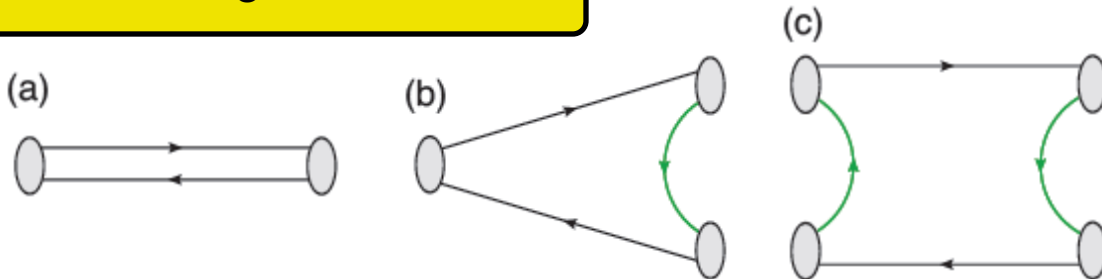
Current spectrum calculations:
no evidence of multi-particle levels

Plot the non-interacting meson levels as a guide

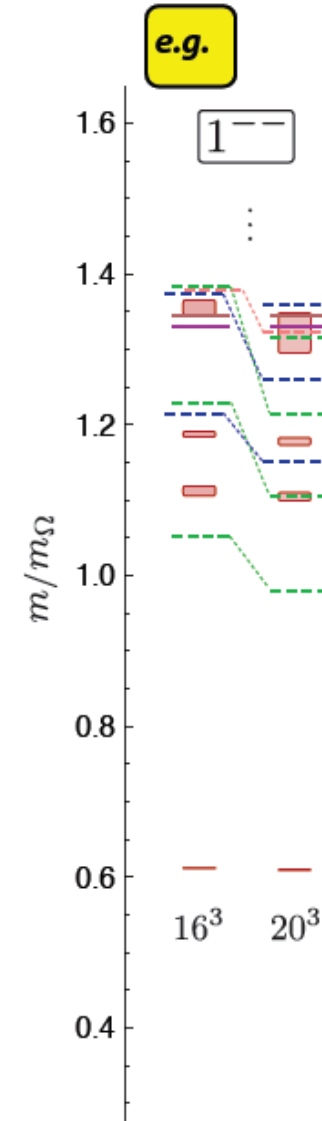
$$|A(\vec{p})B(-\vec{p})\rangle \quad m_{AB} = \sqrt{m_A^2 + \vec{p}^2} + \sqrt{m_B^2 + \vec{p}^2}$$

Require multi-particle operators

- (lattice) helicity construction
- annihilation diagrams



Extract $\delta(E)$ at discrete E

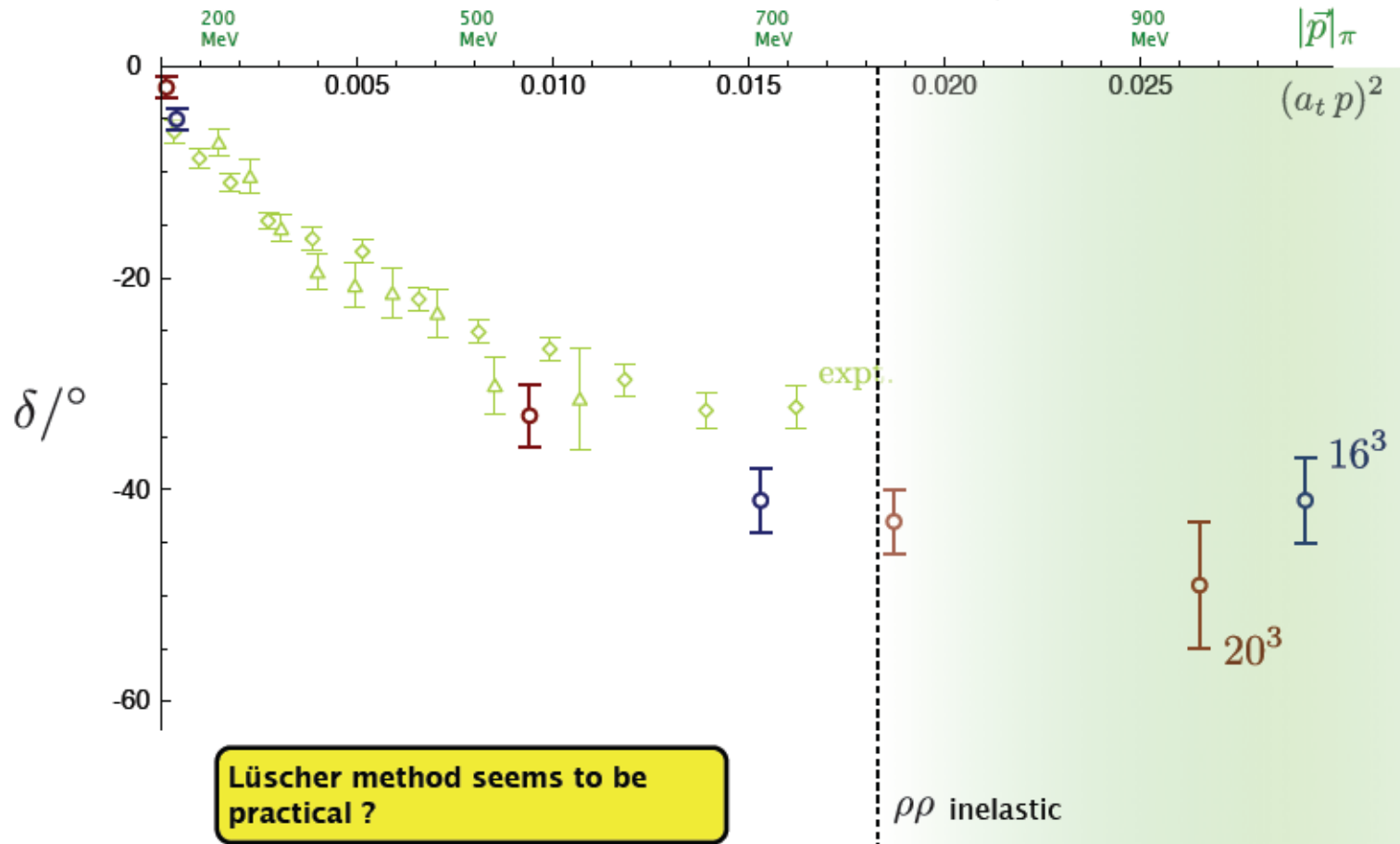


Phase Shifts: demonstration

$\pi\pi$ isospin=2

$N_F = 2+1$ (u,d,s) $m_\pi \sim 400$ MeV

$$\sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} [\bar{\psi}\gamma_5\psi](\vec{x}) \cdot \sum_{\vec{y}} e^{-i(-\vec{p})\cdot\vec{y}} [\bar{\psi}\gamma_5\psi](\vec{y})$$



Prospects

- Strong effort in excited state spectroscopy
 - New operator & correlator constructions → high lying states
 - Finite volume extraction of resonance parameters - promising
- Initial results for excited state spectrum:
 - Suggests baryon spectrum at least as dense as quark model
 - Suggests multiple exotic mesons within range of Hall D
- Resonance determination:
 - Start at heavy masses: have some "elastic scattering"
 - Use larger volumes & smaller pion masses ($m_\pi \sim 230\text{MeV}$)
 - **Now:** multi-particle operators & annihilation diagrams (gpu-s)
 - Need multi-channel finite-volume analysis for (in)elastic scattering
- Future:
 - Transition FF-s, photo-couplings (0803.3020, 0902.2214)
 - Use current insertion probes: TMD's

Backup slides

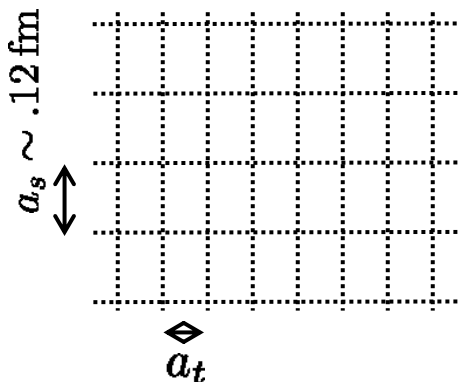
- The end

Regularization of QCD on a lattice

$$Z_{\text{QCD}} = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A_\mu e^{iS[\psi, \bar{\psi}, A_\mu]}$$

quark field gluon field

Discretize on a finite grid - in Euclidean space-time ($t \rightarrow i t$)



$\psi(x) \rightarrow \psi_x$ quark fields on the sites

$A_\mu(x) \rightarrow U_{x,\mu} = e^{-aA_{x\mu}}$ gauge-fields on the links

$$\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U f(\psi, \bar{\psi}, U) e^{-\tilde{S}[\psi, \bar{\psi}, U]}$$



do the fermion integral

$$\int \mathcal{D}U f(Q^{-1}[U], U) \det Q[U] e^{-\tilde{S}_G[U]}$$

probability, $P[U]$



Monte Carlo

$$\sum_{\{U\}} f(Q^{-1}[U], U)$$

$a_t^{-1} \sim 5.6 \text{ GeV}$

$$\left\{ \begin{matrix} 16^3 \\ 20^3 \\ 24^3 \end{matrix} \right\} \times 128$$

$L \gtrsim 2 \text{ fm}$

$$N_F = 2 \oplus 1$$

($u, d \oplus s$)

$230 \text{ MeV} < m_\pi < 700 \text{ MeV}$

Orbital angular momentum via derivatives

Derivatives in ladders:

$$\vec{D}_{l=+1}^{(q)} = \frac{i}{2} (\vec{D}_x + i\vec{D}_y)$$

$$\vec{D}_{l=0}^{(q)} = -\frac{i}{\sqrt{2}} \vec{D}_z$$

$$\vec{D}_{l=-1}^{(q)} = -\frac{i}{2} (\vec{D}_x - i\vec{D}_y)$$

Couple derivatives onto single-site spinors:

$$(D^{[1]}\Psi^{[S]})^{J,M} = \sum_{l,s} \langle 1, l; S, s | J, M \rangle \vec{D}_{L=1,l}^{[1]} \Psi^{S,s}$$

Project onto lattice irreducible representations

$$\mathcal{O}_{\Lambda,\lambda}^{[J]} = \sum_M \mathcal{S}_{\Lambda,\lambda}^{J,M} \mathcal{O}^{J,M}$$

0905.2160 (PRD), 0909.0200 (PRL), 1004.4930

Determining spin on a cubic lattice?

Spin reducible on lattice

$$\text{spin } \frac{5}{2} \text{ (6)} \rightarrow H \text{ (4)} + G_2 \text{ (2)}$$

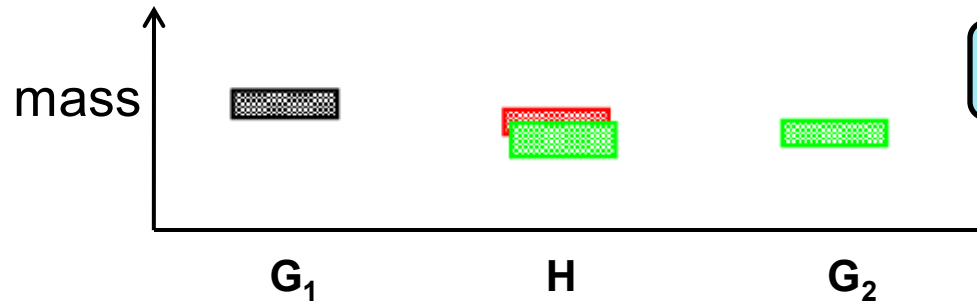
coarse a

finer a

continuum



Might be dynamical degeneracies



Spin 1/2, 3/2, 5/2, or 7/2 ?

Spin reduction & (re)identification

Variational solution:

$$C_{ij}(t) = \langle 0 | \phi_i(t) \phi_j(0) | 0 \rangle \rightarrow \sum_{\alpha} Z_i^{(\alpha)} Z_j^{(\alpha)*} e^{-m_{\alpha} t}$$

Continuum

$$\langle 0 | \mathcal{O} | \frac{5}{2}^{-} (\vec{0}, r) \rangle = Z$$

Lattice



$$\langle 0 | \mathcal{O}_H | \frac{5}{2}^{-} (\vec{0}, r) \rangle = Z_H$$





$$\langle 0 | \mathcal{O}_{G2} | \frac{5}{2}^{-} (\vec{0}, r) \rangle = Z_{G2}$$

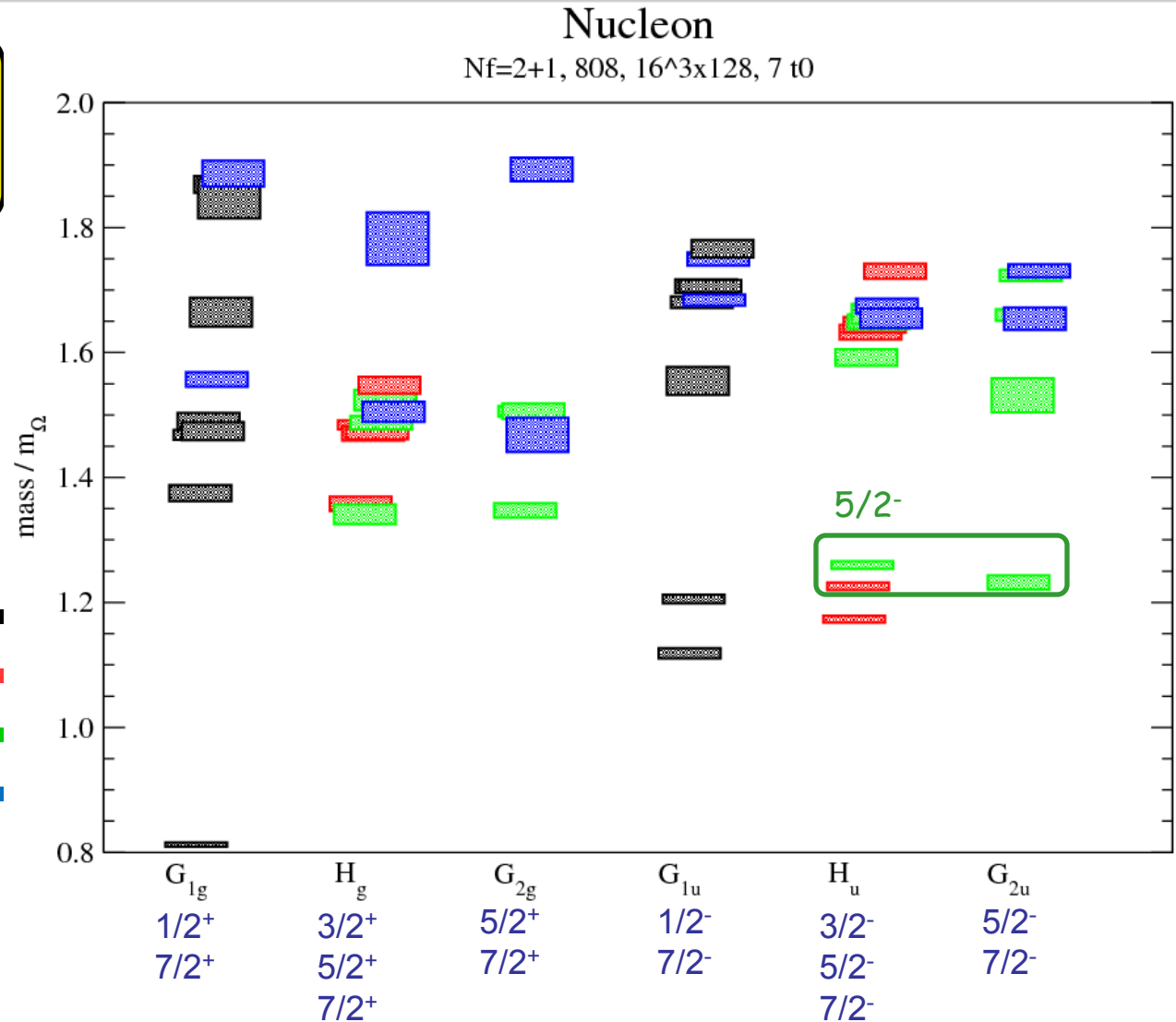
Method: Check if converse is true

Nucleon spectrum in (lattice) group theory

$N_f = 2 + 1$, $m_\pi \sim 580 \text{ MeV}$

Units of Ω baryon

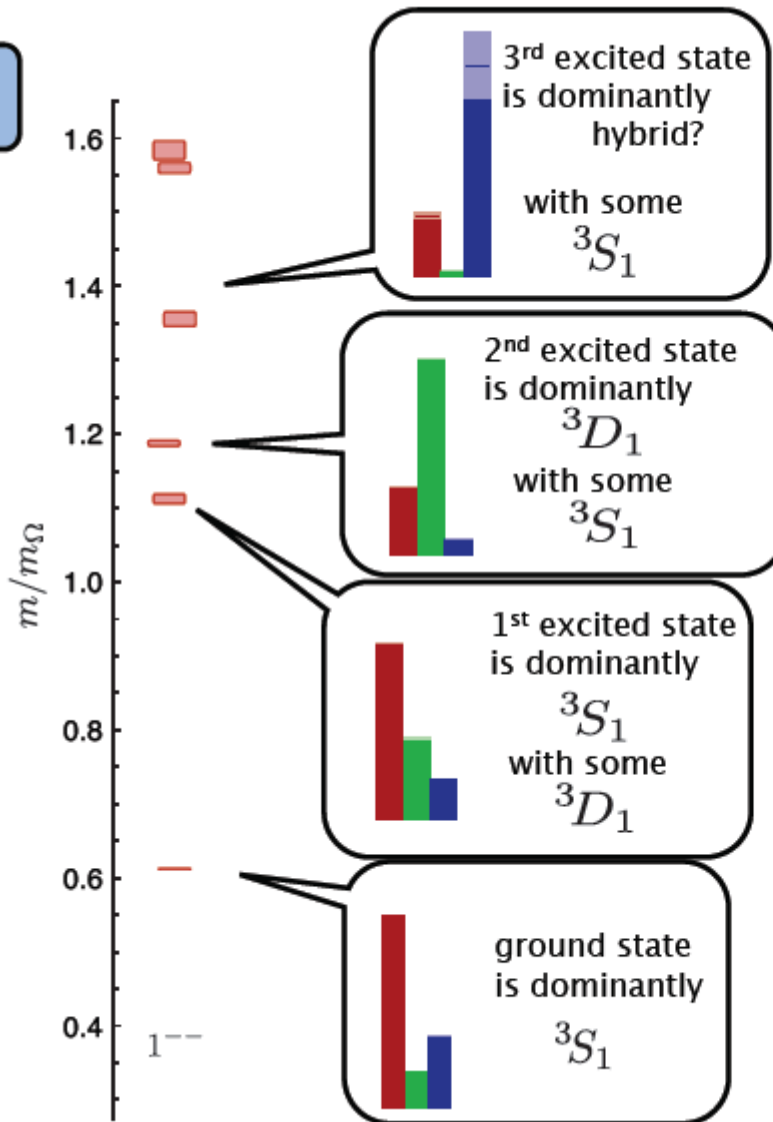
$J = 1/2$ 
 $J = 3/2$ 
 $J = 5/2$ 
 $J = 7/2$ 



PRD 79(2009), PRD 80 (2009), 0909.0200 (PRL)

Interpretation of Meson Spectrum

1⁻⁻⁻



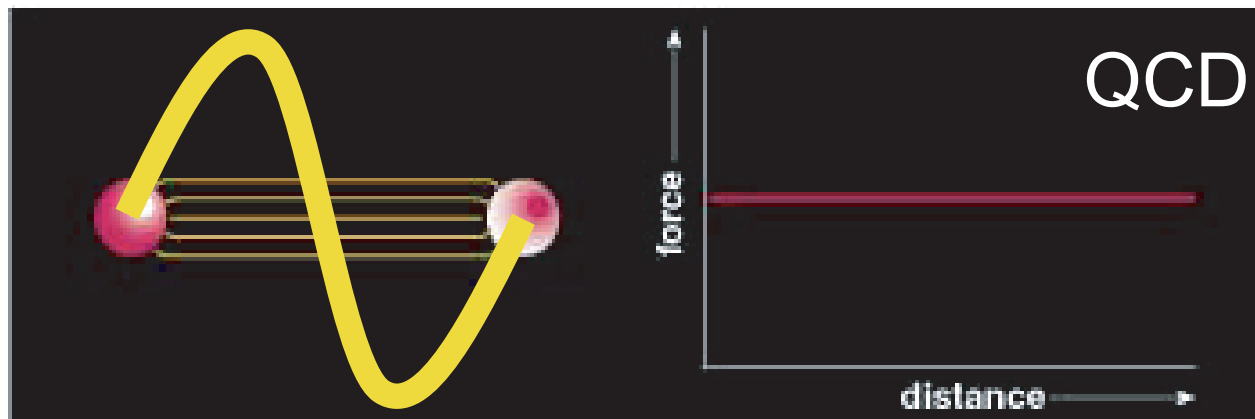
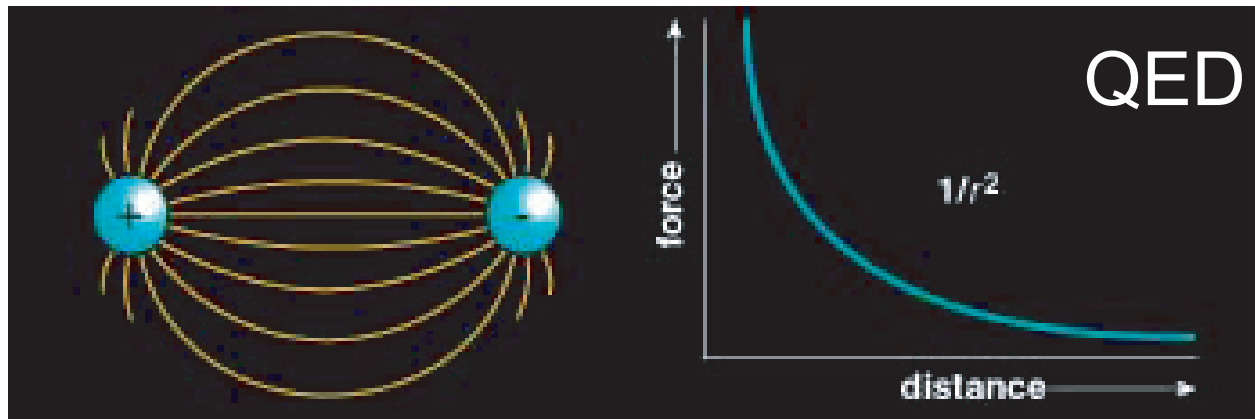
look at the 'overlaps' $Z_n^\Gamma = \langle n | \bar{\psi} \Gamma \psi | 0 \rangle$

ρ 3S_1
 $(\rho \times D_{J=2}^{[2]})^{J=1}$ 3D_1
 $(\pi \times D_{J=1}^{[2]})^{J=1}$ hybrid?

Future: incorporate in bound-state model phenomenology

Future: probe with photon decays

Exotic matter?



Can we observe exotic matter? *Excited string*

Where are the Form Factors??

- Previous efforts
 - Charmonium: excited state E&M transition FF-s (0909.0200)
 - Nucleon: 1st attempt: E&M Roper→N FF-s (0803.3020)
- Spectrum first!
 - Basically have to address "what is a resonance" up front
 - (Simplistic example): FF for a strongly decaying state: linear combination of states

